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DESCRIPTION OF THE PROCESS OF MAGNETIZATION OF HOLLOW CYLINDRICAL RODS FROM SOFT MAGNETIC MATERIALS IN A HOMOGENEOUS QUASI-STATIC MAGNETIC FIELD OF A SOLENOID. HYSTERESIS LOOP

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On the basis of the arctangential approximation of the magnetic hysteresis loop of the material and the expression for the central demagnetization coefficient of hollow cylindrical rods from soft magnetic materials, a method for calculating the hysteresis loops of these rods in a homogeneous quasi-static field of a solenoid is proposed. Calculation and comparison to experimental data of the basic parameters of the magnetic loops hysteresis of hollow cylindrical rods with a different value of the demagnetization coefficient have been made.

Introduction. For solving many theoretical and practical problems of nondestructive testing, the process of magnetization reversal in a homogeneous magnetic field by the magnetic hysteresis loop of bodies of specified dimensions with known magnetic properties of their materials is of interest. For magnetization in a closed magnetic circuit, a number of approximating expressions proposed by different authors [1–4], which are used to describe both the normal magnetization curve and the magnetic hysteresis loop, are known. For a specimen located in a broken magnetic circuit, the most consistent approach to accounting for the influence of the body's demagnetizing field on the shape of the hysteresis loop is the introduction into the approximating expression of the demagnetization coefficient [5].

The aim of the present work is to develop a method for calculating the magnetic hysteresis loop and its basic parameters for a hollow cylindrical rod of given geometry with known magnetic characteristics of the material from which it is made.

Computational Procedure. To describe the magnetic hysteresis loop of the material, let us make use of the previously obtained [4] expression

$$M = \pm \frac{M_s k_{t.s}(H) H_m^2/\pi + k_1 k_3 (H_m) H_c^2}{H_m^2 + k_2 H_m^{3/2} H_c^{1/2} + k_1 H_c^2} \times$$

$$\times \left[2 \arctan \frac{H_c \pm H}{k} - \left(\arctan \frac{H_c + H_m}{k} + \arctan \frac{H_c - H_m}{k} \right) \right], \tag{1}$$

where the signs "+" and "−" refer to the ascending and descending branches of the hysteresis loop;

$$k_{t.s}(H) = \left[\frac{H_{t.s}^2 + \pi H^2}{H_{t.s}^2 + 2H^2} \frac{H_{t.s}^2 + 2H_{m,s}^2}{H_{t.s}^2 + \pi H_{m,s}^2} \right]^{1/2}; \tag{2}$$

$$k = \frac{H_c}{\tan \left(\frac{M_r}{M_s k_{t.s}(0)} \frac{\pi}{2} \right)}; \tag{3}$$

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$$k_1 = \left[\frac{4\pi^2 \left[\frac{1}{M_c} \frac{M_s k_{t.s} (H_c)}{\pi} \arctan\left(2\frac{H_c}{k}\right) - 1 \right]^2 + 1}{4\pi^2 \left[1 - \frac{1}{M_c} k_3 (H_c) \arctan\left(2\frac{H_c}{k}\right)\right]^2 + 1} \right]^{1/2}; \tag{4}$$

$$k_2 = \frac{1}{8} \left\{ \frac{4M_s k_{t.s} (2H_c)/\pi + k_1 k_3 (2H_c)}{M_{2c}} \left[\arctan\left(3\frac{H_c}{k}\right) + \arctan\left(\frac{H_c}{k}\right)\right] - 4 - k_1 \right\}; \tag{5}$$

$$k_3 (H_m) = \frac{\kappa_{in} H_c^2}{k} \frac{1}{2 (H_m^2 + H_c^2)} (k^2 + H_c^2 - H_m^2) \left\{ 1 + \frac{3H_c^3 [k^2 + H_c^2 - H_m^2]}{(H_c^2 + k^2)^2 (H_m^2 + H_c^2)} H_m \right\}. \tag{6}$$

For a hollow cylindrical rod with a demagnetization coefficient $N_{h.cyl}^{cal}$ the following expressions [6] hold:

$$H = H_{ex} - N_{h.cyl}^{cal} M; \tag{7}$$

$$H_m = H_{m,ex} - N_{h.cyl}^{cal} M_{m,h.cyl}; \tag{8}$$

$$\kappa_{in,h.cyl} = \frac{\kappa_{in}}{1 + N_{h.cyl}^{cal}}. \tag{9}$$

Then expressions (1), (2), (6) in view of (7)–(9) will take the form

$$M = \pm \frac{M_s \left[k_{t.s} \left(H_{ex} - N_{h.cyl}^{cal} M \right) \right] \left(H_{m,ex} - N_{h.cyl}^{cal} M_{m,h.cyl} \right)^2 / \pi + k_1 \left[k_3 \left(H_{m,ex} - N_{h.cyl}^{cal} M_{m,h.cyl} \right) \right] H_c^2}{\left(H_{m,ex} - N_{h.cyl}^{cal} M_{m,h.cyl} \right)^2 + k_2 \left(H_{m,ex} - N_{h.cyl}^{cal} M_{m,h.cyl} \right)^{3/2} H_c^{1/2} + k_1 H_c^2}$$

$$\times \left[2 \arctan \frac{H_c \pm \left(H_{ex} - N_{h.cyl}^{cal} M \right)}{k} - \left(\arctan \frac{H_c + H_{m,ex} - N_{h.cyl}^{cal} M_{m,h.cyl}}{k} + \arctan \frac{H_c - H_{m,ex} + N_{h.cyl}^{cal} M_{m,h.cyl}}{k} \right) \right], \tag{10}$$

$$k_{t.s} \left(H_{ex} - N_{h.cyl}^{cal} M \right) = \left[\frac{H_{t.s}^2 + \pi \left(H_{ex} - N_{h.cyl}^{cal} M \right)^2}{H_{t.s}^2 + 2 \left(H_{ex} - N_{h.cyl}^{cal} M \right)^2} \frac{H_{t.s}^2 + 2H_{m,s}^2}{H_{t.s}^2 + \pi H_{m,s}^2} \right]^{1/2}, \tag{11}$$

$$k_3 \left(H_{m,ex} - N_{h.cyl}^{cal} M_{m,h.cyl} \right) = \frac{\kappa_{in}}{1 + N_{h.cyl}^{cal} \kappa_{in}} \frac{H_c^2}{k} \frac{1}{2 \left[\left(H_{m,ex} - N_{h.cyl}^{cal} M_{m,h.cyl} \right)^2 + H_c^2 \right]} \left[k^2 + H_c^2 - \left(H_{m,ex} - N_{h.cyl}^{cal} M_{m,h.cyl} \right)^2 \right]$$

$$\times \left\{ 1 + \frac{2H_c^3 \left[k^2 + H_c^2 - \left(H_{m,ex} - N_{h.cyl}^{cal} M_{m,h.cyl} \right)^2 \right]}{(H_c^2 + k^2)^2 \left[\left(H_{m,ex} - N_{h.cyl}^{cal} M_{m,h.cyl} \right)^2 + H_c^2 \right]} \left(H_{m,ex} - N_{h.cyl}^{cal} M_{m,h.cyl} \right) \right\}. \tag{12}$$

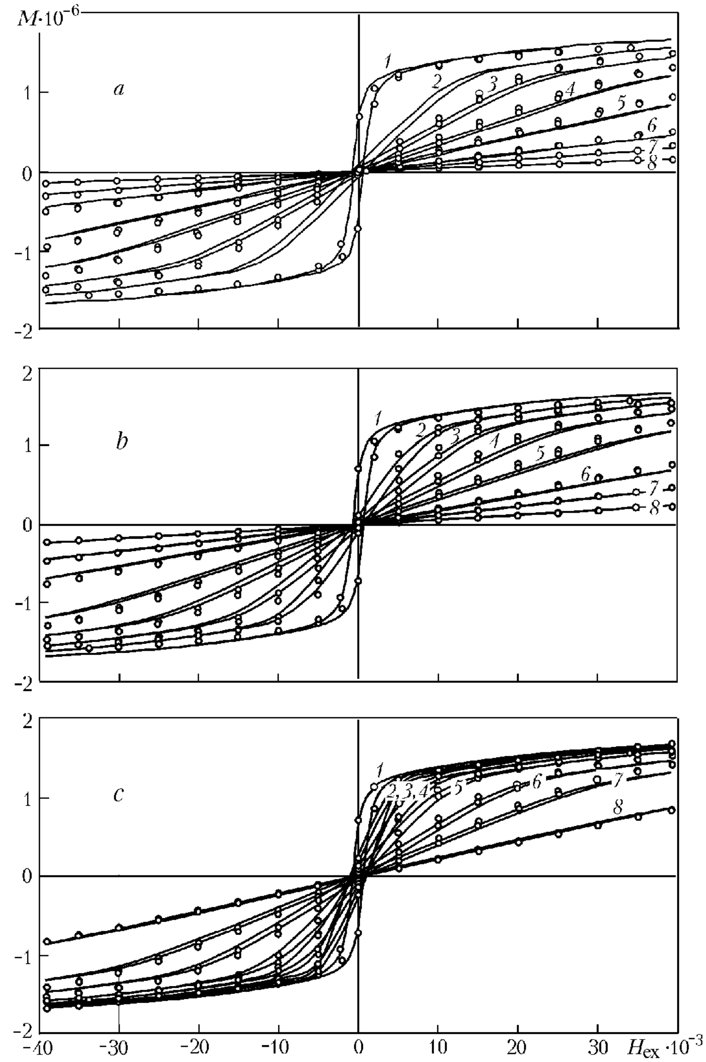

Fig. 1. Magnetic hysteresis loops of the material (1) and hollow cylindrical rods made from it (2–8) at various values of ρ [a) 0.2; b) 0.6; c) 0.9] and λ [1) ∞; 2) 15; 3) 10; 4) 7; 5) 5; 6) 3; 7) 2; 8) 1]. Curves and dots show, respectively, the calculation and the experiment. M, H_{ex}, A/m.

Expression (10) includes the value of the demagnetization coefficient $N^{cal}_{h.cyl}$ of the hollow cylindrical rod. To determine it, let us make use of the formula of the demagnetization coefficient proposed in [7] for hollow cylindrical rods on the assumption that $\mu \to \infty$ for soft magnetic materials:

$$N^{cal}_{h.cyl} = \frac{1-\rho}{2\pi\lambda^3} \int_0^{\lambda} \int_0^{2\pi} \left[\frac{\rho \bar{z}^2 [\gamma\lambda^2 + 2(1-\gamma)\bar{z}^2]}{(\lambda + 1.7\rho)[\rho^2 - \rho(\rho+1)\cos\varphi + 0.25(\rho+1)^2 + \bar{z}^2]^{3/2}} + \frac{\bar{z}^2 [\gamma\lambda^2 + 2(1-\gamma)\bar{z}^2]}{(\lambda + 1.7)[1 - (\rho+1)\cos\varphi + 0.25(\rho+1)^2 + \bar{z}^2]^{3/2}} \right] d\varphi\, d\bar{z}$$
$$+ \frac{0.85\lambda}{\pi(\lambda+1.7)} \int_{\rho}^{1} \int_0^{2\pi} \frac{\bar{r}\, d\varphi\, d\bar{r}}{[\bar{r}^2 - \bar{r}(\rho+1)\cos\varphi + 0.25(\rho+1)^2 + \lambda^2]^{3/2}}, \quad (13)$$

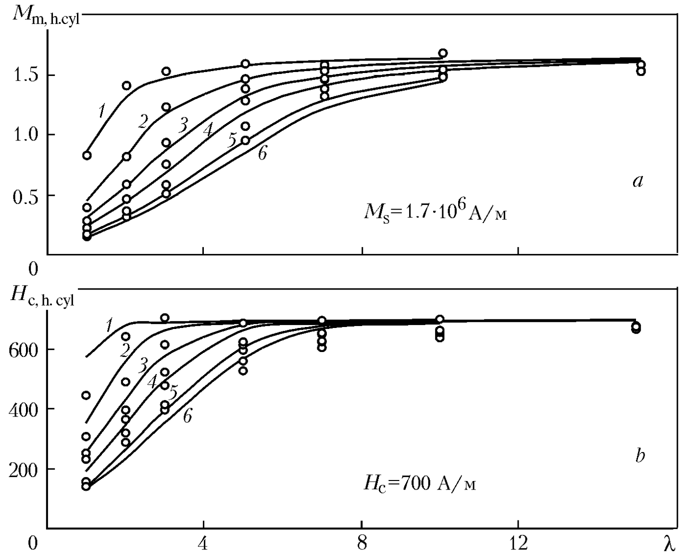

Fig. 2. Maximum magnetization $M_{m,h.cyl}$ (a) and coercive force Hc,h.cyl (b) of hollow cylindrical rods versus their relative length at various values of ρ: 1) 0.9; 2) 0.8; 3) 0.7; 4) 0.6; 5) 0.4; 6) 0.2. Curves and dots show, respectively, the calculation and the experiment. $M_{m,h.cyl}$, $H_{c,h.cyl}$, A/m.

where $\lambda = L/(2R_2)$; $\rho = R_1/R_2$; $\bar{r} = r/R_2$; $\bar{z} = z/R_2$.

Equation (10) representing a transcendental equation for M was solved by the method of sequential approximations. The value of $M_{m,h.cyl}$ was determined from the analogous transcendental equation for the main magnetization curve. The domain of variability of H_{ex} was broken down into a certain number of intervals, and on each of them the initial value of M was given. The computation terminated when the M^i values of the neighboring iterations differed by no more than 0.01%.

Computation and Comparison to Experiment. The major magnetic hysteresis loops were computed for hollow cylindrical rods from steel 45 with an outer radius R_2 = 5 mm; length L equal to 10, 20, 30, 50, 70, 100, and 150 mm; the inner diameter R_1 was varied from 1 to 4.5 mm. For the calculations, we used the following basic parameters of the material determined in a closed magnetic circuit: $M_r = 0.66\cdot10^6$ A/m; H_c = 700 A/m; $M_s = 1.7\cdot10^6$ A/m; $M_c = 2.78\cdot10^5$ A/m; $M_{2c} = 0.65\cdot10^6$ A/m; κ_{in} = 90.

Experimental check of the calculation results was performed by measuring the magnetic characteristics of these same specimens with reversal of their magnetization in a homogeneous quasi-state magnetic field of a solenoid [8]. A magnetization reversing field was furnished by passing through it triangular current of frequency 0.05 Hz. The amplitude of this field $H_{m,ex}$ was 40 kA/m.

Figure 1 shows the hysteresis loops calculated by expression (10) and those obtained experimentally. One can see that they are in good agreement. In both cases, as the relative length λ and the parameter ρ decrease, there is an increase in the slope of the magnetic hysteresis loops. At $\lambda \le 7$ for the thick-walled hollow rod (ρ = 0.2), the hollow rod with $\lambda \le 5$ and ρ = 0.6 and in the case of $\lambda \le 1$ of the thin-walled hollow rod (ρ = 0.9) the loops have the form of practically straight lines.

For many magnetic control methods, the basic information parameters are maximum magnetization M_m, coercive force H_c, residual magnetization M_r, and differential permeability μ_d at $H = H_c$. Therefore, of practical interest is the investigation of the dependence of these magnetic parameters on the dimensions of the body being tested or on the value of their demagnetization coefficient.

The theoretical and experimental results of such investigations for hollow cylindrical rods of different length and different thickness of the wall are presented in Figs. 2 and 3. It is seen that the functional dependences of the basic information parameters of magnetic methods for controlling the structural state of hollow ferromagnetic rods on their dimensions are in good agreement with the dependences of the corresponding parameters determined experimentally.

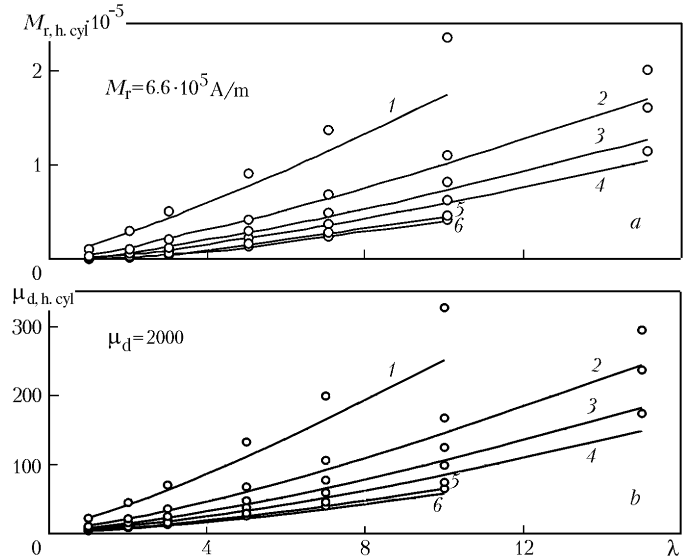

Fig. 3. Residual magnetization $M_{r,h.cyl}$ (a) and differential permeability $\mu_{d,h.cyl}$ (b) as to the hysteresis loop at a field equal to $H_{c,h.cyl}$ of hollow cylindrical rods versus their relative length at various values of ρ: 1) 0.9; 2) 0.8; 3) 0.7; 4) 0.6; 5) 0.4; 6) 0.2. Curves and dots show, respectively, the calculation and the experiment. $M_{r,h.cyl}$, A/m.

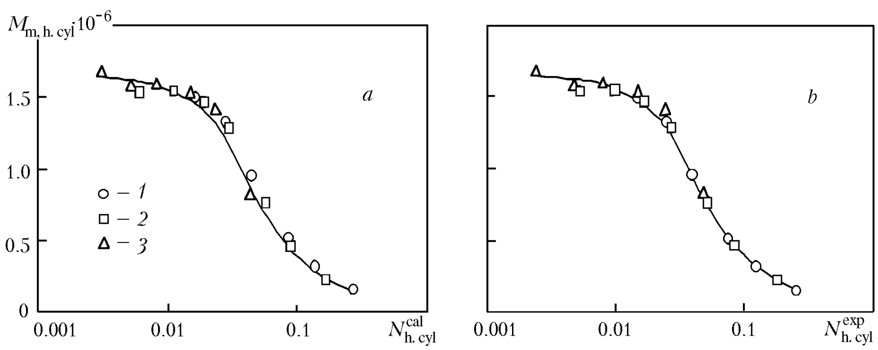

Fig. 4. Maximum magnetization $M_{m,h.cyl}$ versus the calculated (a) and the experimentally measured (b) demagnetization coefficient of the hollow cylindrical rod at various values of ρ: 1) 0.2; 2) 0.6; 3) 0.9. Curves and dots show, respectively, the calculation and the experiment. $M_{m,h.cyl}$, A/m.

A slight difference between the experimental data and the results of the calculation of the maximum magnetization value $M_{m,h.cyl}$ (Fig. 2a) of a hollow cylindrical rod is observed with decreasing parameter ρ. In so doing, technical saturation at $H_{m,ex}$ = 40 kA/m is attained for thin-walled hollow cylindrical rods (ρ = 0.9) already at their relative length $\lambda \geq 5$. With increasing thickness of the wall of the hollow cylindrical rod the value of its relative length, for which at $H_{m,ex}$ = 40 kA/m technical saturation is attained, moves in the direction of larger values. In so doing, the discrepancy between the results of the calculation and the experiment increases. This is due to the fact that in the calculation the change in the demagnetization coefficient along the length of the rod is ignored.

The functional dependence of the coercive force $H_{c,h.cyl}$ of the hollow cylindrical rod on its dimensions (Fig. 2b) is similar in shape to the analogous dependence of the maximum value of magnetization $M_{m,h.cyl}$. And the coercive force $H_{c,h.cyl}$ at $H_{m,ex}$ = 40 kA/m reaches its limiting value equal to H_c at a somewhat smaller relative length and a larger thickness of the wall of hollow cylinders. The widest discrepancy between the calculation results for $H_{c,h.cyl}$ and its experimental values is observed for thin-walled hollow rods in the region of small values of λ ($\lambda < 3$).

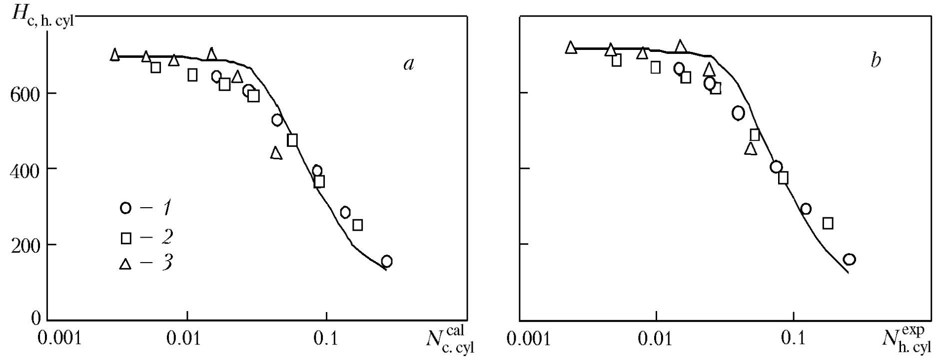

Fig. 5. Coercive force $H_{c,h.cyl}$ versus the calculated (a) and the experimentally measured (b) demagnetization coefficient of the hollow cylindrical rod at various values of ρ: 1) 0.2; 2) 0.6; 3) 0.9. Curves and dots show, respectively, the calculation and the experiment. $H_{c,h.cyl}$, A/m.

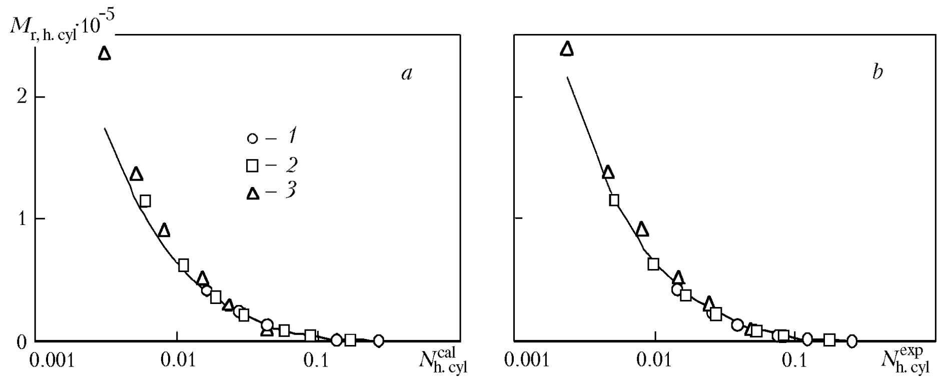

Fig. 6. Residual magnetization $M_{r,h.cyl}$ versus the calculated (a) and the experimentally measured (b) demagnetization coefficient of hollow cylindrical rods at various values of ρ: 1) 0.2; 2) 0.6; 3) 0.9. Curves and dots show, respectively, the calculation and the experiment. $M_{r,h.cyl}$, A/m.

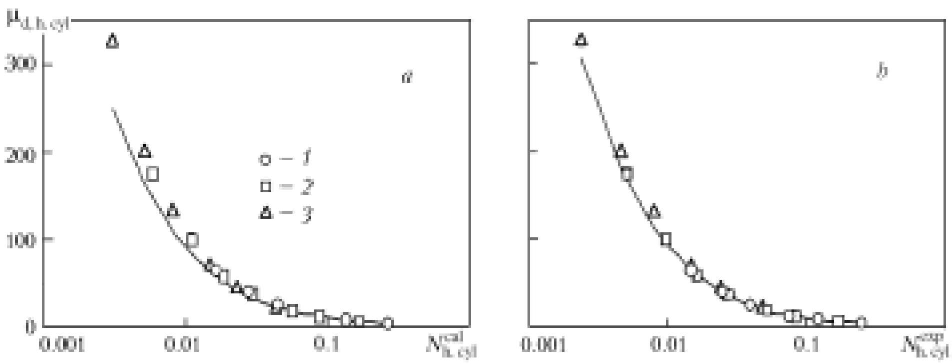

Fig. 7. Differential permeability $\mu_{d,h.cyl}$ with respect to the hysteresis loop at a field equal to $H_{c,h.cyl}$ versus the calculated (a) and the experimentally measured (b) demagnetization coefficient of the hollow cylindrical rod at various values of ρ: 1) 0.2; 2) 0.6; 3) 0.9. Curves and dots show, respectively, the calculation and the experiment. $\mu_{d,h.cyl}$, A/m.

This is explained by the considerably greater change in the internal magnetic field with decreasing relative length λ for thin-walled rods than for rods with a smaller value of ρ.

The residual magnetization $M_{r,h.cyl}$ and the differential permeability $\mu_{d,h.cyl}$ at a field equal to the coercive force increase with increasing relative length of hollow cylinders almost linearly for both thin- and thick-walled hollow rods (Fig. 3) and to a much smaller extent than do the analogous parameters of the rod material. And the difference between the calculation and experimental results widens with increasing relative length and decreasing thickness of the wall of the hollow cylindrical rod, i.e., with increasing value of the demagnetization coefficient. This can be explained by the fact that with decreasing demagnetization coefficient its dependence on the magnetic state of the hollow rod strengthens, which is not taken into account by the formula for calculating $N_{h.cyl}^{cal}$ used by us.

To verify the hypothesis that a considerable part of the error of the calculated parameters of the magnetic hysteresis loop of hollow cylindrical rods is determined by the error of the calculation of the demagnetization coefficient, we calculated the basic parameters of the hysteresis loops of rods with the use of both the experimentally determined demagnetization coefficient $N_{h.cyl}^{exp}$ of hollow rods having different dimensions and the calculated values of the demagnetization coefficient $N_{h.cyl}^{cal}$. The results of these calculations are presented in Figs. 4–7. The lines in the figures show the dependences of the hysteresis loop parameters calculated at various values of ρ.

It is seen from Figs. 4–7 that for the maximum magnetization $M_{m,h.cyl}$ (Fig. 4) and coercive force $H_{c,h.cyl}$ (Fig. 5) of hollow cylindrical rods the difference between the variants of calculation (with the use of $N_{h.cyl}^{cal}$ and $N_{h.cyl}^{exp}$) and the experiment is slight. For the other two parameters $\mu_{d,h.cyl}$ and $M_{r,h.cyl}$ (Figs. 6 and 7) the calculation error is markedly lower in the case where the experimental values of the demagnetization coefficient rather than those calculated by formula (13) are used in the calculations. And this error increases with increasing relative length λ.

Conclusions. The proposed method makes it possible to calculate, with an accuracy acceptable for practical problems, the magnetic hysteresis loop of a hollow cylindrical rod from a soft magnetic material and its parameters, such as maximum magnetization $M_{m,h.cyl}$, coercive force $H_{c,h.cyl}$, residual magnetization $M_{r,h.cyl}$, and differential permeability $\mu_{d,h.cyl}$ at a field equal to the coercive force by the parameters of the normal magnetization curve and the major magnetic hysteresis loop of the rod material (M_s, H_c, M_r, M_c, and M_{2c}) and the dimensions of the rod.

The quantities $M_{m,h.cyl}$ and $H_{c,h.cyl}$ are the least sensitive to a change in the dimensions of a hollow cylindrical rod and reach for thin-walled rods ($\rho = 0.9$) the values of M_s and H_c at a relative length of $\lambda > 5$ and $\lambda > 3$, respectively.

The values of $M_{r,h.cyl}$ and $\mu_{d,h.cyl}$ in the domain of change in the relative length λ of hollow cylindrical rods from 1 to 15 and in the ratio between the inner and outer radii ρ from 0.2 to 0.9 increase with increasing λ almost linearly and to a much lesser extent than do the values of the analogous parameters of the rod material.

NOTATION

H, internal magnetic field strength, A/m; H_c, coercive force of the material as to the major hysteresis loop, A/m; $H_{c,h.cyl}$, coercive force of a hollow cylindrical rod, A/m; H_m, maximum value of the internal magnetic field, A/m; $H_{m,s}$, maximum value of the magnetizing field strength at which M_s was measured, A/m; $H_{t.s} = 32$ kA/m, magnetic field strength at which technical saturation of the material is attained; H_{ex}, external magnetic field strength, A/m; $H_{m,ex}$, maximum value of the external magnetic field, strength, A/m; L, length of a hollow cylindrical rod, m; M, magnetization of the material, A/m; M_c, magnetization of the material on the normal magnetization curve at $H = H_c$, A/m; M_{2c}, magnetization of the material on the normal magnetization curve at $H = 2H_c$, A/m; M_m, maximum magnetization, A/m; $M_{m,h.cyl}$, maximum magnetization value of a hollow cylindrical rod (at a field equal to $H_{m,ex}$), A/m; M_r, residual magnetization of the material as to the major hysteresis loop, A/m; $M_{r,h.cyl}$, residual magnetization of a hollow cylindrical rod, A/m; M_s, saturation magnetization of the material, A/m; $N_{h.cyl}^{cal}$, calculated value of the demagnetization coefficient; $N_{h.cyl}^{exp}$, experimental value of the demagnetization coefficient; r, φ, z, cylindrical coordinates; R_1, inner radius of a hollow cylindrical rod, m; R_2, outer radius of a hollow cylindrical rod, m; γ, constant coefficient equal to 0.8; κ_{in}, initial magnetic susceptibility of the material; $\kappa_{in,h.cyl}$, initial magnetic susceptibility of a hollow cylindrical rod; λ, relative length of a hollow cylindrical rod; μ, permeability; μ_d, differential permeability at $H = H_c$; $\mu_{d,h.cyl}$, differential permeability of a hollow cylindrical rod. Superscripts: i, iteration number;

cal, calculation; exp, experiment. Subscripts: c, coercive; h.cyl, hollow cylindrical rod; m, maximum; s, saturation; t.s, technical saturation; ex, external; *r*, residual; in, initial; d, differential.

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